General Design of Hollow RC Sections under Combined Actions

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1 Abstract

Hollow reinforced concrete sections are consistently considered the preferred solution for medium to large sized bridge projects due to its structural efficiency and the large material savings associated with it.

To fully harvest the structural capacity of hollow sections exposed to combined actions it is necessary to leave behind the simplicity of treating the verification of structural adequacy for normal stresses (beam theory) separately from that of shear stresses (diagonal truss model) and instead fully exploit the advantages of choosing more efficient stress distributions. By exploring the vast possibilities of other statically admissible systems using optimization routines, one will find that longitudinal reinforcement near the neutral axis can be utilized much more efficiently.

In addition, by adhering to the interdependency constraints between normal and shear stresses a much more precise picture of the actual service stress state can be determined. There is therefore the need for a one-step, automated design tool capable of addressing such verifications holistically.

In this paper the theoretical basis and a free to use open-source design tool is presented, allowing for easy access to highly optimized designs capable of pushing the materials to their limits.

Keywords: shear; hollow; design; plasticity; bridge; optimization; membrane.

2 Introduction

The design of large reinforced concrete (RC) hollow sections is recurrent in medium to large sized bridge projects.

Hollow sections are consistently considered the preferred solution for substructure members (piers, towers or columns) and for bridge decks (e.g. prestressed box girders) due to its structural efficiency and the large material savings associated with it.

One of the main challenges with contemporary bridge design is the increasing difficulty in the immediate identification of critical loading scenarios or locations. In fact, as code requirements evolve one needs to account for a growing number of load combinations in the design. In addition, as aesthetic considerations become more predominant, the shapes of structural members become more complex. This challenge is augmented primarily at substructure members, which tend to be exposed to a combination of concurrent force effects (axial load, biaxial flexure, biaxial shear and torsion) together with the absence of refined design methodologies enabling optimal usage of hollow sections' strength without applying a non-linear FE-shell modelling.

In fact, traditional design techniques (e.g. Eurocode, AASHTO) treat the verification of structural adequacy for normal stresses (due to axial load and flexure) using beam theory independently from that for shear stresses (due to shear and torsion) using the diagonal truss model. This simplified two-step approach is inherently conservative as it fails to fully exploit the materials capabilities.

In addition, the diagonal truss model requires the consideration of longitudinal tensile stringers where the normal stress demand is evaluated which fails to utilize the longitudinal reinforcement near the neutral axis.

Alternatively, approaches based on plastic lower bound methods allow for better utilization of the sectional strength. However, the available catalogue of readily applicable solutions is limited to simple cases, such as those of rectangular sections. Moreover, the disregard for strain compatibility in the lower bound solutions renders them generally unsuitable for evaluations related service evaluations.

Therefore, efficient, easy-to-use, automatic verification tools of hollow RC sections under combined actions are required towards a more expedite, general design process.

2.1 Example

For a hollow rectangular section subjected to single components of flexure and shear, the designer would typically assume the bending moment resisted by the top and bottom flanges alone. Determining the corresponding shear flow using Grashof's formula [5] yields the distributions shown in Figure 1. The approach is often simplified even further by neglecting the shear stress in the top and bottom flange and thereby only relying on the webs to resist the shear force. On rare occasions where large bending capacity is needed part of the webs would also be reserved to resist bending.

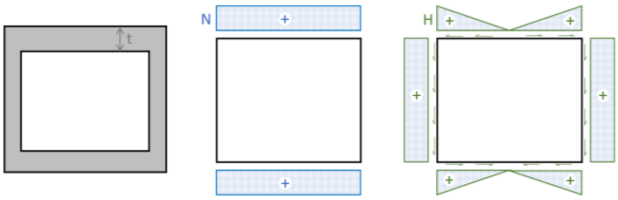

Figure 1: Rectangular hollow cross-section (left) with typically assumed normal flow distribution (center) and corresponding shear flow (right).

If the section is also exposed to a torsional moment the designer would be forced to introduce an additional shear flow in at least two of the walls. Consistently trying to avoid exposing any walls to a combination of shear and normal stress often results in very conservative designs.

Alternatively, one could allow for the combined exposure and now deal with the more complicated flow distributions. Additionally, the designer would have to deal with in-plane membrane verification of all the possible critical combinations using either the reinforcement formulas, implemented in Annex F in Eurocode 2 [4], or the underlying yield conditions presented in [2].

However, this iterative design methodology simply becomes too cumbersome to carry out manually in the design of hollow sections with more complex shape (octagonal, elliptical etc.) or if there are multiple load cases to be considered.

2.2 Objective

A practical, easy-to-use design tool applicable to arbitrary single-cell hollow reinforced concrete sections that addresses the challenges described above has been developed.

One of the paramount goals was that the design tool automatizes the manual procedure of validating the strength demand on the section under combined actions while making use of conventional design code friendly assumptions.

In addition to catering for capacity evaluations it also delivers reliable evaluations pertaining to the performance of the hollow sections under service conditions. The latter addresses a typical limitation of Lower Bound based design approaches where compatibility is disregarded which often leads to additional shell model analysis.

3 Analysis Methodology

The key basic principle is that the hollow cross-section may be considered behaving as a thin-walled flexural member with only in-plane stresses. This assumption simplifies the interdependency between bending and shear behavior such that it can be dealt with efficiently in both Service Limit State (SLS) and Ultimate Limit State (ULS).

3.1 Service Limit State Analysis

Criteria related with fulfilment of compatibility are at the basis for the SLS evaluations. For flexural members this can be carried out in accordance with the well-known Bernoulli's hypothesis which requires that plane sections remain plane. The flexural strain state is then reduced to three variables (e.g. neutral axis angle and two extreme strains) solved attending to global equilibrium with the user-specified sectional forces.

$$N \;= \int_A \sigma_x(y,z)\,dA \tag{1}$$

$$M_y = \int_A \sigma_x(y,z)\cdot(z_{CG}-z)\,dA \tag{2}$$

$$M_z = \int_A \sigma_x(y,z)\cdot(y-y_{CG})\,dA \tag{3}$$

The determination of the shear stress field is not as straight forward as the intention is to accommodate for the post cracking behavior of the section. This renders the Grashof [5] formulation not usable.

The underlying relationship between the normal stresses and the shear stress are shown in **Error! Reference source not found.**, where the local equilibrium of an arbitrary point along the thin walls are schematically represented.

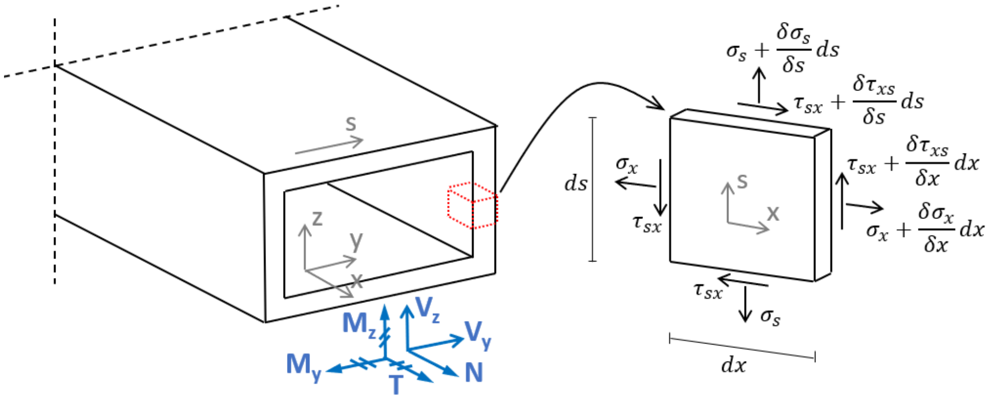

Figure 1. Hollow section with coordinate and loading sign convention including the local equilibrium in thin walled element under plane stress.

From vertical and horizontal equilibrium of the thin-walled element the following equations readily follows:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xs}}{\partial s}=0 \tag{4}$$

$$\frac{\partial\sigma_s}{\partial s}+\frac{\partial\tau_{xs}}{\partial x}=0 \tag{5}$$

The transverse normal stress σ_s (along the perimeter of the hollow section) are generally small and is for simplicity not considered here. Neglecting the transverse normal stress, even though equilibrium in principle requires its presence, is a common assumption in design practice.

Equation (4) gives a direct relationship between the longitudinal gradient of the normal stress and the shear stress gradient in the perimeter direction.

To utilize this relationship, a dual section approach is considered with a second section located at a distance of dx from the reference section is considered. This is a similar strategy as that adopted by Vecchio and Collins in [3].

The marginal difference in the global flexural sectional forces acting on the second section is given from the fundamental relation: $\frac{dM}{dx}=V$.

Once the flexural normal stress distribution of the second section is known, so is the longitudinal gradient $\frac{\partial\sigma_x}{\partial x}$. The shape of the shear stress distribution is then deducted using Equation (4).

Finally, a constant offset of the shear flow distribution is determined (which does not affect the gradients of Equation (5)) using Bredt's formulation, shown in Equation (6), to ensure that the shear stresses also integrates up into the user-specified torsional moment T.

$$\tau_{xs} = \frac{T}{2A_0 t} \qquad (6)$$

where A_0 is the enclosed area within the center line of the walls in the section and t the wall thickness.

This SLS procedure allows for a straightforward evaluation of the expected normal and shear stress distributions even for non-linear constitutive relations where e.g. the concrete is not included if in tension.

At any point an in-plane cracked analysis can be used to convert from the obtained normal and shear stress components to a diagonal concrete compressive stress and two orthogonal reinforcement stresses (see Figure 2).

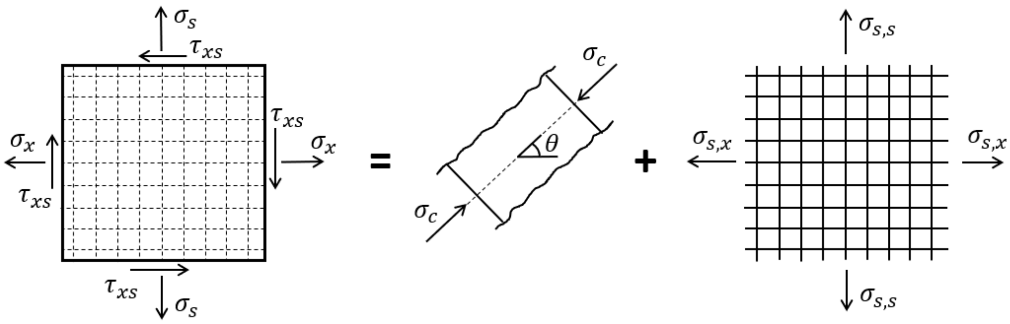

Figure 2. Internal membrane cracked equilibrium.

The cracked equilibrium yields:

$$\sigma_x = \sigma_c \cos^2(\theta) + \rho_{sx}\sigma_{s,x} \qquad (7)$$

$$\sigma_s = \sigma_c \sin^2(\theta) + \rho_{sy}\sigma_{s,s} \qquad (8)$$

$$\tau_{xs} = \sigma_c \cos(\theta)\, sin(\theta) \qquad (9)$$

With $\rho_{si} = A_{si}/t$ where A_{si} is the amount of reinforcement per unit length of corresponding wall.

To reflect the actual stress state the strut angle θ is chosen such that it minimizes the complementary elastic energy in the membrane.

The resulting reinforcement stress level can then be utilized to evaluate if the level of concrete cracking is acceptable. Alternatively, the analysis could be continued with a direct crack width analysis.

3.2 Ultimate Limit State Analysis

For ULS, compatibility no longer needs to be respected and a far greater freedom in selecting an appropriate normal and shear stress distributions is allowed for. According to the Lower Bound Theorem of Plasticity [2] any static admissible stress field within the yield criteria can in principle be considered.

However, the design codes are not clear about the allowable concrete efficiency for flexural effects when one starts to push the normal stress distribution to the extreme.

Instead, a more design code friendly approach is considered where, even in ULS, the normal stresses are determined using the classical plane strain assumption together with a reasonable constitutive relation. While it implements some conservatism in the capacity, it also simplifies the following optimization exercise and yields a flexural capacity more in line with designers' expectations.

The yield conditions considered are those for reinforced membranes as expressed by M.P. Nielsen et al. in [2]. These can be expressed as below for the case where $\sigma_s = 0$:

$$\sigma_x \leq \rho_{sx} f_y \qquad (10)$$

$$\sigma_x \geq -f_c \qquad (11)$$

$$\tau_{xs}^2 \leq (f_{tx} - \sigma_x)\, f_{ts} \;, \qquad f_{ti} = \rho_{si} f_y \qquad (12)$$

$$\tau_{xs}^2 \leq (f_{cx} + \sigma_x)\, f_{cs} \;, \qquad f_{ci} = f_c + \rho_{si} f_y \qquad (13)$$

$$|\tau_{xs}| \leq \tfrac{1}{2} \nu f_c \qquad (14)$$

Note that they are in their form also implemented in the Eurocode 2, where only the last condition

(Equation (14)) includes the concrete efficiency factor ν, typically taken as $0.6(1 - f_c/250)$.

The problem of establishing the load capacity of the hollow section is solved in two basic steps:

1. The Bernoulli's hypothesis is again used to find a static admissible normal stress field, Equations (10, 11).
2. The remaining expressions (12, 13, 14) define the corresponding maximum allowable shear stress at any point along the section walls, the integration of which yields the plastic shear capacity for each of the wall segments. From this a mathematical optimization problem can be formulated: Maximize the shear and torsion load factor λ_S considering the wall shear forces as variables bound by their capacities and constrained by the equilibrium equalities (Equations 15 through 17).

$$V_y = \int_A \tau_{xy}(y,z)\, dA \quad (15)$$

$$V_z = \int_A \tau_{xz}(y,z)\, dA \quad (16)$$

$$T = \int_A [\tau_{xz}(y,z) \cdot y - \tau_{xy}(y,z) \cdot z]\, dA \quad (17)$$

If the solution yields a maximized load factor greater or equal to 1 the sectional capacity is sufficient.

4 Discussion of Design Tool *HollowRC*

The tool successfully deals with the interdependency between the distribution of normal and shear flow in any single-celled hollow cross-section. This is practically impossible to carry out in a reasonable way by manual means due to the overwhelming number of possible suboptimal solutions. Only by implementation of advanced open-source nonlinear optimization algorithms is the tool able to deliver optimized solutions in a highly efficient manner.

The authors established that the mandatory input from the user should be limited to:

- Cross-section geometry, namely wall position and thickness
- Reinforcement ratios for each wall segment
- Material properties
- The combination of sectional forces

In line with the principle of simplicity, the outputs are conveyed in the form of graphical information enabling immediate evaluation of the structural adequacy.

5 Final comments

The design tool is an open-source application freely available and is distributed as a stand-alone executable for Windows to enable wide availability to designers without programming knowledge.

To enhance the adoption process and to allow for other parties to get insight and/or contribute to the further development of the project, the source code is publicly available on GitHub [1], under The GNU General Public License v3.0, together with a user guideline.

6 Conclusions

Enhanced service analysis, capable of better estimating the actual stress state is made possible by addressing the interdependency between normal and shear stress distributions in hollow sections.

Furthermore, the developed *HollowRC* design tool allows for a fully automatic optimization of the possible plastic stress states and thus load carrying capacity, ensuring an optimal usage of the materials.

HollowRC generally allows for a holistic verification of arbitrary single-cell hollow RC sections under combined loading and is made freely available to immediate usage. Despite the tool requiring minimal input, it provides the user with significant data to evaluate the structural adequacy. Moreover, automated optimization algorithms eliminate the need for multiple user choices during the design process alongside ensuring less conservative designs towards material savings and consequent generation of value.

7 Acknowledgements

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